

# 13

## Universal Gravitation

### CHAPTER OUTLINE

- 13.1 Newton's Law of Universal Gravitation
- 13.2 Measuring the Gravitational Constant
- 13.3 Free-Fall Acceleration and the Gravitational Force
- 13.4 Kepler's Laws and the Motion of Planets
- 13.5 The Gravitational Field
- 13.6 Gravitational Potential Energy
- 13.7 Energy Considerations in Planetary and Satellite Motion

### ANSWERS TO QUESTIONS

- Q13.1** Because  $g$  is the same for all objects near the Earth's surface. The larger mass needs a larger force to give it just the same acceleration.
- Q13.2** To a good first approximation, your bathroom scale reading is unaffected because you, the Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weigh slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational field is a little weaker at the center of the Earth than at the surface subsolar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.
- Q13.3** Kepler's second law states that the angular momentum of the Earth is constant as the Earth orbits the sun. Since  $L = m\omega r$ , as the orbital radius decreases from June to December, then the orbital speed must increase accordingly.
- Q13.4** Because both the Earth and Moon are moving in orbit about the Sun. As described by  $F_{\text{gravitational}} = ma_{\text{centripetal}}$ , the gravitational force of the Sun merely keeps the Moon (and Earth) in a nearly circular orbit of radius 150 million kilometers. Because of its velocity, the Moon is kept in its orbit about the Earth by the gravitational force of the Earth. There is no imbalance of these forces, at new moon or full moon.
- Q13.5** Air resistance causes a decrease in the energy of the satellite-Earth system. This reduces the diameter of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of air resistance is to speed up the satellite!
- Q13.6** Kepler's third law, which applies to all planets, tells us that the period of a planet is proportional to  $r^{3/2}$ . Because Saturn and Jupiter are farther from the Sun than Earth, they have longer periods. The Sun's gravitational field is much weaker at a distant Jovian planet. Thus, an outer planet experiences much smaller centripetal acceleration than Earth and has a correspondingly longer period.

**Q13.7** Ten terms are needed in the potential energy:

$$U = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}.$$

With  $N$  particles, you need  $\sum_{i=1}^N (i-1) = \frac{N^2 - N}{2}$  terms.

**Q13.8** No, the escape speed does not depend on the mass of the rocket. If a rocket is launched at escape speed, then the total energy of the rocket-Earth system will be zero. When the separation distance becomes infinite ( $U = 0$ ) the rocket will stop ( $K = 0$ ). In the expression  $\frac{1}{2}mv^2 - \frac{GM_E m}{r} = 0$ , the mass  $m$  of the rocket divides out.

**Q13.9** It takes 100 times more energy for the  $10^5$  kg spacecraft to reach the moon than the  $10^3$  kg spacecraft. Ideally, each spacecraft can reach the moon with zero velocity, so the only term that need be analyzed is the change in gravitational potential energy.  $U$  is proportional to the mass of the spacecraft.

**Q13.10** The escape speed from the Earth is 11.2 km/s and that from the Moon is 2.3 km/s, smaller by a factor of 5. The energy required—and fuel—would be proportional to  $v^2$ , or 25 times more fuel is required to leave the Earth versus leaving the Moon.

**Q13.11** The satellites used for TV broadcast are in geosynchronous orbits. The centers of their orbits are the center of the Earth, and their orbital planes are the Earth's equatorial plane extended. This is the plane of the celestial equator. The communication satellites are so far away that they appear quite close to the celestial equator, from any location on the Earth's surface.

**Q13.12** For a satellite in orbit, one focus of an elliptical orbit, or the center of a circular orbit, must be located at the center of the Earth. If the satellite is over the northern hemisphere for half of its orbit, it must be over the southern hemisphere for the other half. We could share with Easter Island a satellite that would look straight down on Arizona each morning and vertically down on Easter Island each evening.

**Q13.13** The absolute value of the gravitational potential energy of the Earth-Moon system is twice the kinetic energy of the moon relative to the Earth.

**Q13.14** In a circular orbit each increment of displacement is perpendicular to the force applied. The dot product of force and displacement is zero. The work done by the gravitational force on a planet in an elliptical orbit speeds up the planet at closest approach, but negative work is done by gravity and the planet slows as it sweeps out to its farthest distance from the Sun. Therefore, net work in one complete orbit is zero.

**Q13.15** Every point  $q$  on the sphere that does not lie along the axis connecting the center of the sphere and the particle will have companion point  $q'$  for which the components of the gravitational force perpendicular to the axis will cancel. Point  $q'$  can be found by rotating the sphere through  $180^\circ$  about the axis. The forces will not necessarily cancel if the mass is not uniformly distributed, unless the center of mass of the non-uniform sphere still lies along the axis.

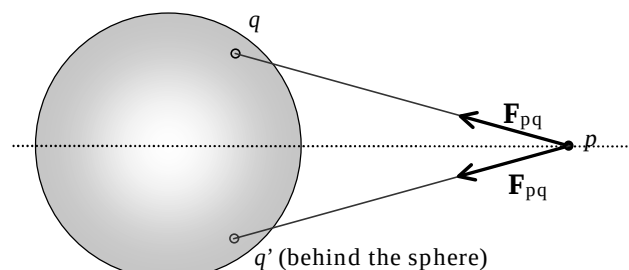


**FIG. Q13.15**

- Q13.16** Speed is maximum at closest approach. Speed is minimum at farthest distance.
- Q13.17** Set the universal description of the gravitational force,  $F_g = \frac{GM_X m}{R_X^2}$ , equal to the local description,  $F_g = ma_{\text{gravitational}}$ , where  $M_X$  and  $R_X$  are the mass and radius of planet  $X$ , respectively, and  $m$  is the mass of a “test particle.” Divide both sides by  $m$ .
- Q13.18** The gravitational force of the Earth on an extra particle at its center must be zero, not infinite as one interpretation of Equation 13.1 would suggest. All the bits of matter that make up the Earth will pull in different outward directions on the extra particle.
- Q13.19** Cavendish determined  $G$ . Then from  $g = \frac{GM}{R^2}$ , one may determine the mass of the Earth.
- Q13.20** The gravitational force is conservative. An encounter with a stationary mass cannot permanently speed up a spacecraft. Jupiter is moving. A spacecraft flying across its orbit just behind the planet will gain kinetic energy as the planet’s gravity does net positive work on it.
- Q13.21** **Method one:** Take measurements from an old kinescope of *Apollo* astronauts on the moon. From the motion of a freely falling object or from the period of a swinging pendulum you can find the acceleration of gravity on the moon’s surface and calculate its mass. **Method two:** One could determine the approximate mass of the moon using an object hanging from an extremely sensitive balance, with knowledge of the position and distance of the moon and the radius of the Earth. First weigh the object when the moon is directly overhead. Then weigh of the object when the moon is just rising or setting. The slight difference between the measured weights reveals the cause of tides in the Earth’s oceans, which is a difference in the strength of the moon’s gravity between different points on the Earth. **Method three:** Much more precisely, from the motion of a spacecraft in orbit around the moon, its mass can be determined from Kepler’s third law.
- Q13.22** The spacecraft did not have enough fuel to stop dead in its high-speed course for the Moon.

## SOLUTIONS TO PROBLEMS

### Section 13.1 Newton’s Law of Universal Gravitation

- P13.1** For two 70-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(70 \text{ kg})(70 \text{ kg})}{(2 \text{ m})^2} = \boxed{\sim 10^{-7} \text{ N}}.$$

- P13.2**  $F = m_1g = \frac{Gm_1m_2}{r^2}$

$$g = \frac{Gm_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(4.00 \times 10^4 \times 10^3 \text{ kg})}{(100 \text{ m})^2} = \boxed{2.67 \times 10^{-7} \text{ m/s}^2}$$

- P13.3** (a) At the midpoint between the two objects, the forces exerted by the 200-kg and 500-kg objects are oppositely directed,

and from  $F_g = \frac{Gm_1m_2}{r^2}$

we have  $\sum F = \frac{G(50.0 \text{ kg})(500 \text{ kg} - 200 \text{ kg})}{(0.200 \text{ m})^2} = \boxed{2.50 \times 10^{-5} \text{ N}}$  toward the 500-kg object.

- (b) At a point between the two objects at a distance  $d$  from the 500-kg objects, the net force on the 50.0-kg object will be zero when

$$\frac{G(50.0 \text{ kg})(200 \text{ kg})}{(0.400 \text{ m} - d)^2} = \frac{G(50.0 \text{ kg})(500 \text{ kg})}{d^2}$$

or  $d = \boxed{0.245 \text{ m}}$

**P13.4**  $m_1 + m_2 = 5.00 \text{ kg}$        $m_2 = 5.00 \text{ kg} - m_1$

$$F = G \frac{m_1 m_2}{r^2} \Rightarrow 1.00 \times 10^{-8} \text{ N} = \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right) \frac{m_1 (5.00 \text{ kg} - m_1)}{(0.200 \text{ m})^2}$$

$$(5.00 \text{ kg})m_1 - m_1^2 = \frac{(1.00 \times 10^{-8} \text{ N})(0.0400 \text{ m}^2)}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2} = 6.00 \text{ kg}^2$$

Thus,  $m_1^2 - (5.00 \text{ kg})m_1 + 6.00 \text{ kg} = 0$

or  $(m_1 - 3.00 \text{ kg})(m_1 - 2.00 \text{ kg}) = 0$

giving  $\boxed{m_1 = 3.00 \text{ kg, so } m_2 = 2.00 \text{ kg}}$ . The answer  $m_1 = 2.00 \text{ kg}$  and  $m_2 = 3.00 \text{ kg}$  is physically equivalent.

- P13.5** The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by

$$\begin{aligned} \mathbf{F}_{24} &= G \frac{m_4 m_2}{r_{24}^2} \hat{\mathbf{j}} = \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right) \frac{(4.00 \text{ kg})(2.00 \text{ kg})}{(3.00 \text{ m})^2} \hat{\mathbf{j}} \\ &= 5.93 \times 10^{-11} \hat{\mathbf{j}} \text{ N} \end{aligned}$$

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left

$$\begin{aligned} \mathbf{F}_{64} &= G \frac{m_4 m_6}{r_{64}^2} (-\hat{\mathbf{i}}) = \left( -6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right) \frac{(4.00 \text{ kg})(6.00 \text{ kg})}{(4.00 \text{ m})^2} \hat{\mathbf{i}} \\ &= -10.0 \times 10^{-11} \hat{\mathbf{i}} \text{ N} \end{aligned}$$

Therefore, the resultant force on the 4.00-kg mass is  $\mathbf{F}_4 = \mathbf{F}_{24} + \mathbf{F}_{64} = \boxed{(-10.0 \hat{\mathbf{i}} + 5.93 \hat{\mathbf{j}}) \times 10^{-11} \text{ N}}$ .

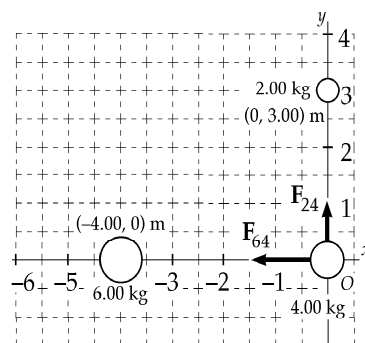


**FIG. P13.5**

- P13.6** (a) The Sun-Earth distance is  $1.496 \times 10^{11}$  m and the Earth-Moon distance is  $3.84 \times 10^8$  m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496 \times 10^{11} \text{ m} - 3.84 \times 10^8 \text{ m} = 1.492 \times 10^{11} \text{ m}$$

The mass of the Sun, Earth, and Moon are

$$M_S = 1.99 \times 10^{30} \text{ kg}$$

$$M_E = 5.98 \times 10^{24} \text{ kg}$$

and

$$M_M = 7.36 \times 10^{22} \text{ kg}$$

We have  $F_{SM} = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30})(7.36 \times 10^{22})}{(1.492 \times 10^{11})^2} = \boxed{4.39 \times 10^{20} \text{ N}}$

(b)  $F_{EM} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24})(7.36 \times 10^{22})}{(3.84 \times 10^8)^2} = \boxed{1.99 \times 10^{20} \text{ N}}$

(c)  $F_{SE} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30})(5.98 \times 10^{24})}{(1.496 \times 10^{11})^2} = \boxed{3.55 \times 10^{22} \text{ N}}$

Note that the force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

## Section 13.2 Measuring the Gravitational Constant

**P13.7**  $F = \frac{GMm}{r^2} = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \frac{(1.50 \text{ kg})(15.0 \times 10^{-3} \text{ kg})}{(4.50 \times 10^{-2} \text{ m})^2} = \boxed{7.41 \times 10^{-10} \text{ N}}$

**P13.8** Let  $\theta$  represent the angle each cable makes with the vertical,  $L$  the cable length,  $x$  the distance each ball scrunches in, and  $d = 1$  m the original distance between them. Then  $r = d - 2x$  is the separation of the balls. We have

$$\sum F_y = 0: \quad T \cos \theta - mg = 0$$

$$\sum F_x = 0: \quad T \sin \theta - \frac{Gmm}{r^2} = 0$$

$$\text{Then} \quad \tan \theta = \frac{Gmm}{r^2 mg} \quad \frac{x}{\sqrt{L^2 - x^2}} = \frac{Gm}{g(d - 2x)^2} \quad x(d - 2x)^2 = \frac{Gm}{g} \sqrt{L^2 - x^2}.$$

The factor  $\frac{Gm}{g}$  is numerically small. There are two possibilities: either  $x$  is small or else  $d - 2x$  is small.

**Possibility one:** We can ignore  $x$  in comparison to  $d$  and  $L$ , obtaining

$$x(1 \text{ m})^2 = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(100 \text{ kg})}{(9.8 \text{ m/s}^2)} 45 \text{ m} \quad x = 3.06 \times 10^{-8} \text{ m}.$$

The separation distance is  $r = 1 \text{ m} - 2(3.06 \times 10^{-8} \text{ m}) = \boxed{1.000 \text{ m} - 61.3 \text{ nm}}$ .

**Possibility two:** If  $d - 2x$  is small,  $x \approx 0.5$  m and the equation becomes

$$(0.5 \text{ m})r^2 = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(100 \text{ kg})}{(9.8 \text{ N/kg})} \sqrt{(45 \text{ m})^2 - (0.5 \text{ m})^2} \quad r = \boxed{2.74 \times 10^{-4} \text{ m}}.$$

For this answer to apply, the spheres would have to be compressed to a density like that of the nucleus of atom.

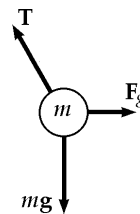


FIG. P13.8

### Section 13.3 Free-Fall Acceleration and the Gravitational Force

**P13.9**  $a = \frac{MG}{(4R_E)^2} = \frac{9.80 \text{ m/s}^2}{16.0} = \boxed{0.613 \text{ m/s}^2}$  toward the Earth.

**P13.10**  $g = \frac{GM}{R^2} = \frac{G\rho\left(\frac{4\pi R^3}{3}\right)}{R^2} = \frac{4}{3}\pi G\rho R$

If  $\frac{g_M}{g_E} = \frac{1}{6} = \frac{\frac{4\pi G\rho_M R_M}{3}}{\frac{4\pi G\rho_E R_E}{3}}$

then  $\frac{\rho_M}{\rho_E} = \left(\frac{g_M}{g_E}\right)\left(\frac{R_E}{R_M}\right) = \left(\frac{1}{6}\right)(4) = \boxed{\frac{2}{3}}.$

**P13.11** (a) At the zero-total field point,  $\frac{GmM_E}{r_E^2} = \frac{GmM_M}{r_M^2}$

so  $r_M = r_E \sqrt{\frac{M_M}{M_E}} = r_E \sqrt{\frac{7.36 \times 10^{22}}{5.98 \times 10^{24}}} = \frac{r_E}{9.01}$

$$r_E + r_M = 3.84 \times 10^8 \text{ m} = r_E + \frac{r_E}{9.01}$$

$$r_E = \frac{3.84 \times 10^8 \text{ m}}{1.11} = \boxed{3.46 \times 10^8 \text{ m}}$$

(b) At this distance the acceleration due to the Earth's gravity is

$$g_E = \frac{GM_E}{r_E^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(3.46 \times 10^8 \text{ m})^2}$$

$$g_E = \boxed{3.34 \times 10^{-3} \text{ m/s}^2 \text{ directed toward the Earth}}$$

### Section 13.4 Kepler's Laws and the Motion of Planets

**P13.12** (a)  $v = \frac{2\pi r}{T} = \frac{2\pi(384\,400) \times 10^3 \text{ m}}{27.3 \times (86\,400 \text{ s})} = \boxed{1.02 \times 10^3 \text{ m/s}}.$

(b) In one second, the Moon falls a distance

$$x = \frac{1}{2}at^2 = \frac{1}{2} \frac{v^2}{r} t^2 = \frac{1}{2} \frac{(1.02 \times 10^3)^2}{(3.844 \times 10^8)} \times (1.00)^2 = 1.35 \times 10^{-3} \text{ m} = \boxed{1.35 \text{ mm}}.$$

The Moon only moves inward 1.35 mm for every 1020 meters it moves along a straight-line path.

**P13.13** Applying Newton's 2nd Law,  $\sum F = ma$  yields  $F_g = ma_c$  for each star:

$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \quad \text{or} \quad M = \frac{4v^2 r}{G}.$$

We can write  $r$  in terms of the period,  $T$ , by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so  $2\pi r = vT$ . Therefore

$$M = \frac{4v^2 r}{G} = \frac{4v^2}{G} \left( \frac{vT}{2\pi} \right)$$

$$\text{so, } M = \frac{2v^3 T}{\pi G} = \frac{2(220 \times 10^3 \text{ m/s})^3 (14.4 \text{ d})(86\,400 \text{ s/d})}{\pi(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)} = \boxed{1.26 \times 10^{32} \text{ kg} = 63.3 \text{ solar masses}}$$

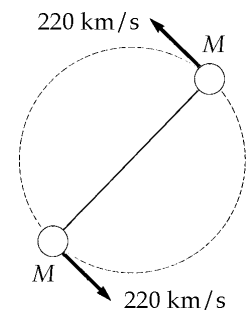


**FIG. P13.13**

- P13.14** Since speed is constant, the distance traveled between  $t_1$  and  $t_2$  is equal to the distance traveled between  $t_3$  and  $t_4$ . The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

$$\text{So } \frac{1}{2}bv(t_2 - t_1) = \frac{1}{2}bv(t_4 - t_3)$$

states that the particle's radius vector sweeps out equal areas in equal times.

- P13.15**  $T^2 = \frac{4\pi^2 a^3}{GM}$  (Kepler's third law with  $m \ll M$ )

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2 (4.22 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.77 \times 86\,400 \text{ s})^2} = \boxed{1.90 \times 10^{27} \text{ kg}}$$

(Approximately 316 Earth masses)

- P13.16** By conservation of angular momentum for the satellite,

$$r_p v_p = r_a v_a \quad \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2\,289 \text{ km} + 6.37 \times 10^3 \text{ km}}{459 \text{ km} + 6.37 \times 10^3 \text{ km}} = \frac{8\,659 \text{ km}}{6\,829 \text{ km}} = \boxed{1.27}.$$

We do not need to know the period.

- P13.17** By Kepler's Third Law,  $T^2 = ka^3$  ( $a$  = semi-major axis)

For any object orbiting the Sun, with  $T$  in years and  $a$  in A.U.,  $k = 1.00$ . Therefore, for Comet Halley

$$(75.6)^2 = (1.00) \left( \frac{0.570 + y}{2} \right)^3$$

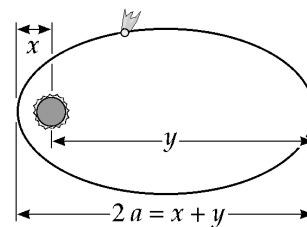


FIG. P13.17

The farthest distance the comet gets from the Sun is

$$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}} \quad (\text{out around the orbit of Pluto})$$

- P13.18**  $\sum F = ma:$   $\frac{Gm_{\text{planet}}M_{\text{star}}}{r^2} = \frac{m_{\text{planet}}v^2}{r}$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^3 = r_x^3\omega_x^2 = r_y^3\omega_y^2$$

$$\omega_y = \omega_x \left( \frac{r_x}{r_y} \right)^{3/2} \quad \omega_y = \left( \frac{90.0^\circ}{5.00 \text{ yr}} \right)^{3/2} = \frac{468^\circ}{5.00 \text{ yr}}$$

So  $\boxed{\text{planet Y has turned through 1.30 revolutions}}.$

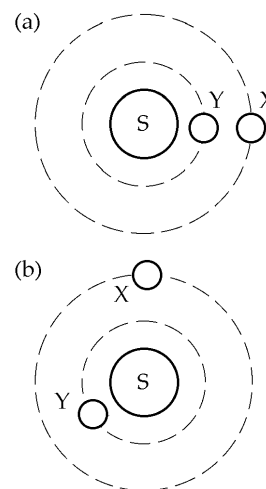


FIG. P13.18

$$\text{P13.19} \quad \frac{GM_J}{(R_J + d)^2} = \frac{4\pi^2(R_J + d)}{T^2}$$

$$GM_J T^2 = 4\pi^2(R_J + d)^3$$

$$(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.90 \times 10^{27} \text{ kg})(9.84 \times 3600)^2 = 4\pi^2(6.99 \times 10^7 + d)^3$$

$$d = \boxed{8.92 \times 10^7 \text{ m}} = \boxed{89\,200 \text{ km}} \text{ above the planet}$$

**P13.20** The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal force:

$$\frac{GM_s m}{R_s^2} = \frac{mv^2}{R_s} = mR_s \omega^2$$

$$\text{so } \omega = \sqrt{\frac{GM_s}{R_s^3}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)[2(1.99 \times 10^{30} \text{ kg})]}{(10.0 \times 10^3 \text{ m})^3}}$$

$$\omega = \boxed{1.63 \times 10^4 \text{ rad/s}}$$

**\*P13.21** The speed of a planet in a circular orbit is given by

$$\sum F = ma: \quad \frac{GM_{\text{sun}} m}{r^2} = \frac{mv^2}{r} \quad v = \sqrt{\frac{GM_{\text{sun}}}{r}}.$$

$$\text{For Mercury the speed is } v_M = \sqrt{\frac{(6.67 \times 10^{-11})(1.99 \times 10^{30}) \text{ m}^2}{(5.79 \times 10^{10}) \text{ s}^2}} = 4.79 \times 10^4 \text{ m/s}$$

$$\text{and for Pluto, } v_P = \sqrt{\frac{(6.67 \times 10^{-11})(1.99 \times 10^{30}) \text{ m}^2}{(5.91 \times 10^{12}) \text{ s}^2}} = 4.74 \times 10^3 \text{ m/s}.$$

With greater speed, Mercury will eventually move farther from the Sun than Pluto. With original distances  $r_P$  and  $r_M$  perpendicular to their lines of motion, they will be equally far from the Sun after time  $t$  where

$$\sqrt{r_P^2 + v_P^2 t^2} = \sqrt{r_M^2 + v_M^2 t^2}$$

$$r_P^2 - r_M^2 = (v_M^2 - v_P^2)t^2$$

$$t = \sqrt{\frac{(5.91 \times 10^{12} \text{ m})^2 - (5.79 \times 10^{10} \text{ m})^2}{(4.79 \times 10^4 \text{ m/s})^2 - (4.74 \times 10^3 \text{ m/s})^2}} = \sqrt{\frac{3.49 \times 10^{25} \text{ m}^2}{2.27 \times 10^9 \text{ m}^2/\text{s}^2}} = 1.24 \times 10^8 \text{ s} = \boxed{393 \text{ yr}}.$$

**\*P13.22** For the Earth,  $\sum F = ma$ :  $\frac{GM_s m}{r^2} = \frac{mv^2}{r} = \frac{m}{r} \left( \frac{2\pi r}{T} \right)^2$ .

Then  $GM_s T^2 = 4\pi^2 r^3$ .

Also the angular momentum  $L = mvr = m \frac{2\pi r}{T} r$  is a constant for the Earth.

We eliminate  $r = \sqrt{\frac{LT}{2\pi m}}$  between the equations:

$$GM_s T^2 = 4\pi^2 \left( \frac{LT}{2\pi m} \right)^{3/2}$$

$$GM_s T^{1/2} = 4\pi^2 \left( \frac{L}{2\pi m} \right)^{3/2}$$

Now the rate of change is described by

$$GM_s \left( \frac{1}{2} T^{-1/2} \frac{dT}{dt} \right) + G \left( 1 \frac{dM_s}{dt} T^{1/2} \right) = 0 \quad \frac{dT}{dt} = - \frac{dM_s}{dt} \left( 2 \frac{T}{M_s} \right) \approx \frac{\Delta T}{T}$$

$$\Delta T \approx -\Delta t \frac{dM_s}{dt} \left( 2 \frac{T}{M_s} \right) = -5\,000 \text{ yr} \left( \frac{3.16 \times 10^7 \text{ s}}{1 \text{ yr}} \right) \left( -3.64 \times 10^9 \text{ kg/s} \right) \left( 2 \frac{1 \text{ yr}}{1.991 \times 10^{30} \text{ kg}} \right)$$

$$\Delta T = \boxed{1.82 \times 10^{-2} \text{ s}}$$

## Section 13.5 The Gravitational Field

**P13.23**  $\mathbf{g} = \frac{Gm}{l^2} \hat{\mathbf{i}} + \frac{Gm}{l^2} \hat{\mathbf{j}} + \frac{Gm}{2l^2} (\cos 45.0^\circ \hat{\mathbf{i}} + \sin 45.0^\circ \hat{\mathbf{j}})$

so  $\mathbf{g} = \frac{GM}{l^2} \left( 1 + \frac{1}{2\sqrt{2}} \right) (\hat{\mathbf{i}} + \hat{\mathbf{j}})$  or

$$\mathbf{g} = \boxed{\frac{Gm}{l^2} \left( \sqrt{2} + \frac{1}{2} \right) \text{ toward the opposite corner}}$$

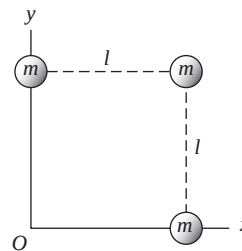


FIG. P13.23

**P13.24** (a)  $F = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) [100(1.99 \times 10^{30} \text{ kg})(10^3 \text{ kg})]}{(1.00 \times 10^4 \text{ m} + 50.0 \text{ m})^2} = \boxed{1.31 \times 10^{17} \text{ N}}$

(b)  $\Delta F = \frac{GMm}{r_{\text{front}}^2} - \frac{GMm}{r_{\text{back}}^2}$

$$\Delta g = \frac{\Delta F}{m} = \frac{GM(r_{\text{back}}^2 - r_{\text{front}}^2)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

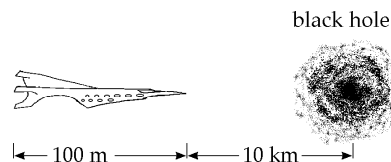


FIG. P13.24

$$\Delta g = \frac{(6.67 \times 10^{-11}) [100(1.99 \times 10^{30})] [(1.01 \times 10^4 \text{ m})^2 - (1.00 \times 10^4 \text{ m})^2]}{(1.00 \times 10^4 \text{ m})^2 (1.01 \times 10^4 \text{ m})^2}$$

$$\Delta g = \boxed{2.62 \times 10^{12} \text{ N/kg}}$$

**P13.25**  $g_1 = g_2 = \frac{MG}{r^2 + a^2}$   
 $g_{1y} = -g_{2y}$   $g_y = g_{1y} + g_{2y}$   
 $g_{1x} = g_{2x} = g_2 \cos \theta$   $\cos \theta = \frac{r}{(a^2 + r^2)^{1/2}}$   
 $\mathbf{g} = 2g_{2x}(-\hat{\mathbf{i}})$   
 or  $\mathbf{g} = \frac{2MGr}{(r^2 + a^2)^{3/2}}$  toward the center of mass

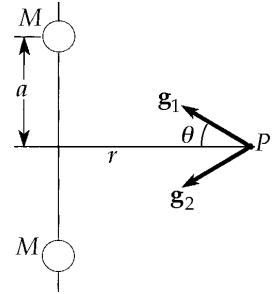


FIG. P13.25

## Section 13.6 Gravitational Potential Energy

**P13.26** (a)  $U = -\frac{GM_E m}{r} = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(100 \text{ kg})}{(6.37 + 2.00) \times 10^6 \text{ m}} = \boxed{-4.77 \times 10^9 \text{ J}}.$

(b), (c) Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F = \frac{GM_E m}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(100 \text{ kg})}{(8.37 \times 10^6 \text{ m})^2} = \boxed{569 \text{ N}}$$

**P13.27**  $U = -G \frac{Mm}{r}$  and  $g = \frac{GM_E}{R_E^2}$   
 so that  $\Delta U = -GMm \left( \frac{1}{3R_E} - \frac{1}{R_E} \right) = \frac{2}{3} mgR_E$   
 $\Delta U = \frac{2}{3} (1000 \text{ kg})(9.80 \text{ m/s}^2)(6.37 \times 10^6 \text{ m}) = \boxed{4.17 \times 10^{10} \text{ J}}.$

**P13.28** The height attained is not small compared to the radius of the Earth, so  $U = mgy$  does not apply;  $U = -\frac{GM_1 M_2}{r}$  does. From launch to apogee at height  $h$ ,

$$\begin{aligned} K_i + U_i + \Delta E_{\text{mch}} &= K_f + U_f: \quad \frac{1}{2} M_p v_i^2 - \frac{GM_E M_p}{R_E} + 0 = 0 - \frac{GM_E M_p}{R_E + h} \\ \frac{1}{2} (10.0 \times 10^3 \text{ m/s})^2 - (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \left( \frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m}} \right) \\ &= - \left( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \right) \left( \frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m} + h} \right) \\ (5.00 \times 10^7 \text{ m}^2/\text{s}^2) - (6.26 \times 10^7 \text{ m}^2/\text{s}^2) &= \frac{-3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{6.37 \times 10^6 \text{ m} + h} \\ 6.37 \times 10^6 \text{ m} + h &= \frac{3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{1.26 \times 10^7 \text{ m}^2/\text{s}^2} = 3.16 \times 10^7 \text{ m} \\ \boxed{h = 2.52 \times 10^7 \text{ m}} \end{aligned}$$

$$\text{P13.29} \quad (a) \quad \rho = \frac{M_S}{\frac{4}{3}\pi r_E^2} = \frac{3(1.99 \times 10^{30} \text{ kg})}{4\pi(6.37 \times 10^6 \text{ m})^3} = \boxed{1.84 \times 10^9 \text{ kg/m}^3}$$

$$(b) \quad g = \frac{GM_S}{r_E^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}{(6.37 \times 10^6 \text{ m})^2} = \boxed{3.27 \times 10^6 \text{ m/s}^2}$$

$$(c) \quad U_g = -\frac{GM_S m}{r_E} = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})(1.00 \text{ kg})}{6.37 \times 10^6 \text{ m}} = \boxed{-2.08 \times 10^{13} \text{ J}}$$

$$\text{P13.30} \quad W = -\Delta U = -\left(\frac{-Gm_1 m_2}{r} - 0\right)$$

$$W = \frac{(+6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.36 \times 10^{22} \text{ kg})(1.00 \times 10^3 \text{ kg})}{1.74 \times 10^6 \text{ m}} = \boxed{2.82 \times 10^9 \text{ J}}$$

$$\text{P13.31} \quad (a) \quad U_{\text{Tot}} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1 m_2}{r_{12}}\right)$$

$$U_{\text{Tot}} = -\frac{3(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.00 \times 10^{-3} \text{ kg})^2}{0.300 \text{ m}} = \boxed{-1.67 \times 10^{-14} \text{ J}}$$

(b) At the center of the equilateral triangle

\*P13.32 (a) Energy conservation of the object-Earth system from release to radius  $r$ :

$$(K + U_g)_{\text{altitude } h} = (K + U_g)_{\text{radius } r}$$

$$0 - \frac{GM_E m}{R_E + h} = \frac{1}{2}mv^2 - \frac{GM_E m}{r}$$

$$v = \left( 2GM_E \left( \frac{1}{r} - \frac{1}{R_E + h} \right) \right)^{1/2} = -\frac{dr}{dt}$$

$$(b) \quad \int_i^f dt = \int_i^f -\frac{dr}{v} = \int_f^i \frac{dr}{v}. \text{ The time of fall is}$$

$$\Delta t = \int_{R_E}^{R_E + h} \left( 2GM_E \left( \frac{1}{r} - \frac{1}{R_E + h} \right) \right)^{-1/2} dr$$

$$\Delta t = \int_{6.37 \times 10^6 \text{ m}}^{6.87 \times 10^6 \text{ m}} \left[ 2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24} \left( \frac{1}{r} - \frac{1}{6.87 \times 10^6 \text{ m}} \right) \right]^{-1/2} dr$$

We can enter this expression directly into a mathematical calculation program.

Alternatively, to save typing we can change variables to  $u = \frac{r}{10^6}$ . Then

$$\Delta t = (7.977 \times 10^{14})^{-1/2} \int_{6.37}^{6.87} \left( \frac{1}{10^6 u} - \frac{1}{6.87 \times 10^6} \right)^{-1/2} 10^6 du = 3.541 \times 10^{-8} \frac{10^6}{(10^6)^{-1/2}} \int_{6.37}^{6.87} \left( \frac{1}{u} - \frac{1}{6.87} \right)^{-1/2} du$$

A mathematics program returns the value 9.596 for this integral, giving for the time of fall  $\Delta t = 3.541 \times 10^{-8} \times 10^9 \times 9.596 = 339.8 = \boxed{340 \text{ s}}$ .

## Section 13.7 Energy Considerations in Planetary and Satellite Motion

**P13.33**  $\frac{1}{2}mv_i^2 + GM_E m \left( \frac{1}{r_f} - \frac{1}{r_i} \right) = \frac{1}{2}mv_f^2$   $\frac{1}{2}v_i^2 + GM_E \left( 0 - \frac{1}{R_E} \right) = \frac{1}{2}v_f^2$

or  $v_f^2 = v_i^2 - \frac{2GM_E}{R_E}$

and  $v_f = \left( v_i^2 - \frac{2GM_E}{R_E} \right)^{1/2}$

$$v_f = \left[ \left( 2.00 \times 10^4 \right)^2 - 1.25 \times 10^8 \right]^{1/2} = \boxed{1.66 \times 10^4 \text{ m/s}}$$

**P13.34** (a)  $v_{\text{solar escape}} = \sqrt{\frac{2M_{\text{Sun}}G}{R_{E\text{Sun}}}} = \boxed{42.1 \text{ km/s}}$

(b) Let  $r = R_{E\text{Sun}}x$  represent variable distance from the Sun, with  $x$  in astronomical units.

$$v = \sqrt{\frac{2M_{\text{Sun}}G}{R_{E\text{Sun}}x}} = \frac{42.1}{\sqrt{x}}$$

If  $v = \frac{125\,000 \text{ km}}{3\,600 \text{ s}}$ , then  $x = 1.47 \text{ A.U.} = \boxed{2.20 \times 10^{11} \text{ m}}$

(at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape).

**P13.35** To obtain the orbital velocity, we use  $\sum F = \frac{mMG}{R^2} = \frac{mv^2}{R}$

or  $v = \sqrt{\frac{MG}{R}}$

We can obtain the escape velocity from  $\frac{1}{2}mv_{\text{esc}}^2 = \frac{mMG}{R}$

or  $v_{\text{esc}} = \sqrt{\frac{2MG}{R}} = \boxed{\sqrt{2}v}$

**P13.36**  $\frac{v_i^2}{R_E + h} = \frac{GM_E}{(R_E + h)^2}$

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2} \left( \frac{GM_E m}{R_E + h} \right) = \frac{1}{2} \left[ \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(500 \text{ kg})}{(6.37 \times 10^6 \text{ m}) + (0.500 \times 10^6 \text{ m})} \right] = 1.45 \times 10^{10} \text{ J}$$

The change in gravitational potential energy of the satellite-Earth system is

$$\begin{aligned} \Delta U &= \frac{GM_E m}{R_i} - \frac{GM_E m}{R_f} = GM_E m \left( \frac{1}{R_i} - \frac{1}{R_f} \right) \\ &= (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(500 \text{ kg})(-1.14 \times 10^{-8} \text{ m}^{-1}) = -2.27 \times 10^9 \text{ J} \end{aligned}$$

Also,  $K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(500 \text{ kg})(2.00 \times 10^3 \text{ m/s})^2 = 1.00 \times 10^9 \text{ J}.$

The energy transformed due to friction is

$$\Delta E_{\text{int}} = K_i - K_f - \Delta U = (1.45 - 1.00 + 2.27) \times 10^9 \text{ J} = \boxed{1.58 \times 10^{10} \text{ J}}.$$

**394** Universal Gravitation

**P13.37**  $F_c = F_G$  gives  $\frac{mv^2}{r} = \frac{GmM_E}{r^2}$

which reduces to  $v = \sqrt{\frac{GM_E}{r}}$

and period  $= \frac{2\pi r}{v} = 2\pi r \sqrt{\frac{r}{GM_E}}$ .

(a)  $r = R_E + 200 \text{ km} = 6\,370 \text{ km} + 200 \text{ km} = 6\,570 \text{ km}$

Thus,

$$\begin{aligned} \text{period} &= 2\pi(6.57 \times 10^6 \text{ m}) \sqrt{\frac{(6.57 \times 10^6 \text{ m})}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} \\ T &= 5.30 \times 10^3 \text{ s} = 88.3 \text{ min} = \boxed{1.47 \text{ h}} \end{aligned}$$

(b)  $v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.57 \times 10^6 \text{ m})}} = \boxed{7.79 \text{ km/s}}$

(c)  $K_f + U_f = K_i + U_i$  + energy input, gives

$$\text{input} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \left(\frac{-GM_E m}{r_f}\right) - \left(\frac{-GM_E m}{r_i}\right) \quad (1)$$

$$r_i = R_E = 6.37 \times 10^6 \text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400 \text{ s}} = 4.63 \times 10^2 \text{ m/s}$$

Substituting the appropriate values into (1) yields the

$$\text{minimum energy input} = \boxed{6.43 \times 10^9 \text{ J}}$$

**P13.38** The gravitational force supplies the needed centripetal acceleration.

$$\text{Thus, } \frac{GM_E m}{(R_E + h)^2} = \frac{mv^2}{(R_E + h)} \quad \text{or} \quad v^2 = \frac{GM_E}{R_E + h}$$

$$(a) \quad T = \frac{2\pi r}{v} = \frac{2\pi(R_E + h)}{\sqrt{\frac{GM_E}{R_E + h}}} \quad T = \boxed{2\pi \sqrt{\frac{(R_E + h)^3}{GM_E}}}$$

$$(b) \quad v = \boxed{\sqrt{\frac{GM_E}{R_E + h}}}$$

$$(c) \quad \text{Minimum energy input is} \quad \Delta E_{\min} = (K_f + U_{gf}) - (K_i - U_{gi}).$$

It is simplest to

launch the satellite from a location on the equator, and launch it toward the east.

$$\text{This choice has the object starting with energy} \quad K_i = \frac{1}{2}mv_i^2$$

$$\text{with} \quad v_i = \frac{2\pi R_E}{1.00 \text{ day}} = \frac{2\pi R_E}{86\,400 \text{ s}} \quad \text{and} \quad U_{gi} = -\frac{GM_E m}{R_E}.$$

$$\text{Thus,} \quad \Delta E_{\min} = \frac{1}{2}m \left( \frac{GM_E}{R_E + h} \right) - \frac{GM_E m}{R_E + h} - \frac{1}{2}m \left[ \frac{4\pi^2 R_E^2}{(86\,400 \text{ s})^2} \right] + \frac{GM_E m}{R_E}$$

$$\text{or} \quad \Delta E_{\min} = \boxed{GM_E m \left[ \frac{R_E + 2h}{2R_E(R_E + h)} \right] - \frac{2\pi^2 R_E^2 m}{(86\,400 \text{ s})^2}}$$

$$\mathbf{P13.39} \quad E_{\text{tot}} = -\frac{GMm}{2r}$$

$$\Delta E = \frac{GMm}{2} \left( \frac{1}{r_i} - \frac{1}{r_f} \right) = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{2} \frac{10^3 \text{ kg}}{10^3 \text{ m}} \left( \frac{1}{6\,370 + 100} - \frac{1}{6\,370 + 200} \right)$$

$$\Delta E = 4.69 \times 10^8 \text{ J} = \boxed{469 \text{ MJ}}$$

$$\mathbf{P13.40} \quad g_E = \frac{Gm_E}{r_E^2} \quad g_U = \frac{Gm_U}{r_U^2}$$

$$(a) \quad \frac{g_U}{g_E} = \frac{m_U r_E^2}{m_E r_U^2} = 14.0 \left( \frac{1}{3.70} \right)^2 = 1.02 \quad g_U = (1.02)(9.80 \text{ m/s}^2) = \boxed{10.0 \text{ m/s}^2}$$

$$(b) \quad v_{\text{esc},E} = \sqrt{\frac{2Gm_E}{r_E}}; \quad v_{\text{esc},U} = \sqrt{\frac{2Gm_U}{r_U}}: \quad \frac{v_{\text{esc},E}}{v_{\text{esc},U}} = \sqrt{\frac{m_U r_E}{m_E r_U}} = \sqrt{\frac{14.0}{3.70}} = 1.95$$

For the Earth, from the text's table of escape speeds,  $v_{\text{esc},E} = 11.2 \text{ km/s}$

$$\therefore v_{\text{esc},U} = (1.95)(11.2 \text{ km/s}) = \boxed{21.8 \text{ km/s}}$$

**P13.41** The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 m_2 (1.495 \times 10^{23} \text{ kg})}{\text{kg}^2 (2.64 \times 10^6 \text{ m})}$$

$$U_1 = -3.78 \times 10^6 m_2 \text{ m}^2/\text{s}^2$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 m_2 (1.90 \times 10^{27} \text{ kg})}{\text{kg}^2 (1.071 \times 10^9 \text{ m})}$$

$$U_2 = -1.18 \times 10^8 m_2 \text{ m}^2/\text{s}^2$$

To escape from both requires

$$\frac{1}{2} m_2 v_{\text{esc}}^2 = + (3.78 \times 10^6 + 1.18 \times 10^8) m_2 \text{ m}^2/\text{s}^2$$

$$v_{\text{esc}} = \sqrt{2 \times 1.22 \times 10^8 \text{ m}^2/\text{s}^2} = \boxed{15.6 \text{ km/s}}$$

**P13.42** We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the Universe:

$$\frac{1}{2} m v_{\text{esc}}^2 = \frac{GM_m m}{R_m}$$

$$v_{\text{esc}} = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_{\text{launch}} = 2 \sqrt{\frac{2GM_m}{R_m}}$$

Now for the flight from moon to Earth

$$(K + U)_i = (K + U)_f$$

$$\frac{1}{2} m v_{\text{launch}}^2 - \frac{GmM_m}{R_m} - \frac{GmM_E}{r_{\text{el}}} = \frac{1}{2} m v_{\text{impact}}^2 - \frac{GmM_m}{r_{m_2}} - \frac{GmM_E}{R_E}$$

$$\frac{4GM_m}{R_m} - \frac{GM_m}{R_m} - \frac{GM_E}{r_{\text{el}}} = \frac{1}{2} v_{\text{impact}}^2 - \frac{GM_m}{r_{m_2}} - \frac{GM_E}{R_E}$$

$$v_{\text{impact}} = \left[ 2G \left( \frac{3M_m}{R_m} + \frac{M_m}{r_{m_2}} + \frac{M_E}{R_E} - \frac{M_E}{r_{\text{el}}} \right) \right]^{1/2}$$

$$= \left[ 2G \left( \frac{3 \times 7.36 \times 10^{22} \text{ kg}}{1.74 \times 10^6 \text{ m}} + \frac{7.36 \times 10^{22} \text{ kg}}{3.84 \times 10^8 \text{ m}} + \frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m}} - \frac{5.98 \times 10^{24} \text{ kg}}{3.84 \times 10^8 \text{ m}} \right) \right]^{1/2}$$

$$= \left[ 2G (1.27 \times 10^{17} + 1.92 \times 10^{14} + 9.39 \times 10^{17} - 1.56 \times 10^{16}) \text{ kg/m} \right]^{1/2}$$

$$= \left[ 2 (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) 10.5 \times 10^{17} \text{ kg/m} \right]^{1/2} = \boxed{11.8 \text{ km/s}}$$

- \*P13.43** (a) Energy conservation for the object-Earth system from firing to apex:

$$(K + U_g)_i = (K + U_g)_f$$

$$\frac{1}{2}mv_i^2 - \frac{GmM_E}{R_E} = 0 - \frac{GmM_E}{R_E + h}$$

where  $\frac{1}{2}mv_{\text{esc}}^2 = \frac{GmM_E}{R_E}$ . Then

$$\frac{1}{2}v_i^2 - \frac{1}{2}v_{\text{esc}}^2 = -\frac{1}{2}v_{\text{esc}}^2 \frac{R_E}{R_E + h}$$

$$v_{\text{esc}}^2 - v_i^2 = \frac{v_{\text{esc}}^2 R_E}{R_E + h}$$

$$\frac{1}{v_{\text{esc}}^2 - v_i^2} = \frac{R_E + h}{v_{\text{esc}}^2 R_E}$$

$$h = \frac{v_{\text{esc}}^2 R_E}{v_{\text{esc}}^2 - v_i^2} - R_E = \frac{v_{\text{esc}}^2 R_E - v_{\text{esc}}^2 R_E + v_i^2 R_E}{v_{\text{esc}}^2 - v_i^2}$$

$$h = \frac{R_E v_i^2}{v_{\text{esc}}^2 - v_i^2}$$

(b)  $h = \frac{6.37 \times 10^6 \text{ m} (8.76)^2}{(11.2)^2 - (8.76)^2} = \boxed{1.00 \times 10^7 \text{ m}}$

- (c) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation

$$v_i^2 = v_{\text{esc}}^2 \left( 1 - \frac{R_E}{R_E + h} \right) = v_{\text{esc}}^2 \left( \frac{h}{R_E + h} \right) = (11.2 \times 10^3 \text{ m/s})^2 \left( \frac{2.51 \times 10^7 \text{ m}}{6.37 \times 10^6 \text{ m} + 2.51 \times 10^7 \text{ m}} \right)$$

$$= 1.00 \times 10^8 \text{ m}^2/\text{s}^2$$

$$v_i = \boxed{1.00 \times 10^4 \text{ m/s}}$$

- (d) With  $v_i \ll v_{\text{esc}}$ ,  $h \approx \frac{R_E v_i^2}{v_{\text{esc}}^2} = \frac{R_E v_i^2 R_E}{2GM_E}$ . But  $g = \frac{GM_E}{R_E^2}$ , so  $h = \frac{v_i^2}{2g}$ , in agreement with
- $$0^2 = v_i^2 + 2(-g)(h - 0).$$

- P13.44** For a satellite in an orbit of radius  $r$  around the Earth, the total energy of the satellite-Earth system is  $E = -\frac{GM_E m}{2r}$ . Thus, in changing from a circular orbit of radius  $r = 2R_E$  to one of radius  $r = 3R_E$ , the required work is

$$W = \Delta E = -\frac{GM_E m}{2r_f} + \frac{GM_E m}{2r_i} = GM_E m \left[ \frac{1}{4R_E} - \frac{1}{6R_E} \right] = \boxed{\frac{GM_E m}{12R_E}}.$$

**\*P13.45** (a) The major axis of the orbit is  $2a = 50.5 \text{ AU}$  so  $a = 25.25 \text{ AU}$   
 Further, in Figure 13.5,  $a + c = 50 \text{ AU}$  so  $c = 24.75 \text{ AU}$   
 Then  $e = \frac{c}{a} = \frac{24.75}{25.25} = \boxed{0.980}$

(b) In  $T^2 = K_s a^3$  for objects in solar orbit, the Earth gives us

$$(1 \text{ yr})^2 = K_s (1 \text{ AU})^3 \quad K_s = \frac{(1 \text{ yr})^2}{(1 \text{ AU})^3}$$

Then  $T^2 = \frac{(1 \text{ yr})^2}{(1 \text{ AU})^3} (25.25 \text{ AU})^3 \quad T = \boxed{127 \text{ yr}}$

(c)  $U = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.991 \times 10^{30} \text{ kg})(1.2 \times 10^{10} \text{ kg})}{50(1.496 \times 10^{11} \text{ m})} = \boxed{-2.13 \times 10^{17} \text{ J}}$

**\*P13.46** (a) For the satellite  $\sum F = ma \quad \frac{GmM_E}{r^2} = \frac{mv_0^2}{r}$

$$\boxed{v_0 = \left( \frac{GM_E}{r} \right)^{1/2}}$$

(b) Conservation of momentum in the forward direction for the exploding satellite:

$$\begin{aligned} (\sum mv)_i &= (\sum mv)_f \\ 5mv_0 &= 4mv_i + m0 \\ v_i &= \frac{5}{4}v_0 = \boxed{\frac{5}{4} \left( \frac{GM_E}{r} \right)^{1/2}} \end{aligned}$$

(c) With velocity perpendicular to radius, the orbiting fragment is at perigee. Its apogee distance and speed are related to  $r$  and  $v_i$  by  $4mr v_i = 4mr_f v_f$  and

$$\frac{1}{2} 4mv_i^2 - \frac{GM_E 4m}{r} = \frac{1}{2} 4mv_f^2 - \frac{GM_E 4m}{r_f}. \text{ Substituting } v_f = \frac{v_i r}{r_f} \text{ we have}$$

$$\frac{1}{2} v_i^2 - \frac{GM_E}{r} = \frac{1}{2} \frac{v_i^2 r^2}{r_f^2} - \frac{GM_E}{r_f}. \text{ Further, substituting } v_i^2 = \frac{25}{16} \frac{GM_E}{r} \text{ gives}$$

$$\begin{aligned} \frac{25}{32} \frac{GM_E}{r} - \frac{GM_E}{r} &= \frac{25}{32} \frac{GM_E r}{r_f^2} - \frac{GM_E}{r_f} \\ \frac{-7}{32r} &= \frac{25r}{32r_f^2} - \frac{1}{r_f} \end{aligned}$$

Clearing of fractions,  $-7r_f^2 = 25r^2 - 32rr_f$  or  $7\left(\frac{r_f}{r}\right)^2 - 32\left(\frac{r_f}{r}\right) + 25 = 0$  giving

$$\frac{r_f}{r} = \frac{+32 \pm \sqrt{32^2 - 4(7)(25)}}{14} = \frac{50}{14} \text{ or } \frac{14}{14}. \text{ The latter root describes the starting point. The outer}$$

$$\text{end of the orbit has } \frac{r_f}{r} = \frac{25}{7}; \quad \boxed{\frac{r_f}{r} = \frac{25}{7}}.$$

## Additional Problems

**P13.47** Let  $m$  represent the mass of the spacecraft,  $r_E$  the radius of the Earth's orbit, and  $x$  the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of  $F_s = \frac{GM_s m}{(r_E - x)^2}$

while the Earth exerts on it a radial outward force of  $F_E = \frac{GM_E m}{x^2}$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus, 
$$F_s - F_E = \frac{GM_s m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)} \left[ \frac{2\pi(r_E - x)}{T} \right]^2$$

which reduces to 
$$\frac{GM_s}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2}. \quad (1)$$

Cleared of fractions, this equation would contain powers of  $x$  ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that  $x$  is between  $1.47 \times 10^9$  m and  $1.48 \times 10^9$  m by substituting both of these as trial solutions, along with the following data:  $M_s = 1.991 \times 10^{30}$  kg,  $M_E = 5.983 \times 10^{24}$  kg,  $r_E = 1.496 \times 10^{11}$  m, and  $T = 1.000$  yr =  $3.156 \times 10^7$  s.

With  $x = 1.47 \times 10^9$  m substituted into equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

or  $5.868 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$

With  $x = 1.48 \times 10^9$  m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.8708 \times 10^{-3} \text{ m/s}^2$$

or  $5.8709 \times 10^{-3} \text{ m/s}^2 \approx 5.8708 \times 10^{-3} \text{ m/s}^2$ .

Since the first trial solution makes the left-hand side of equation (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is  $1.48 \times 10^9$  m.

As an equation of fifth degree, equation (1) has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by  $60^\circ$ . Plans are under way to gain perspective on the Sun by placing a spacecraft at one of these two co-orbital Lagrange points. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

**P13.48** The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by

$$a = G \frac{M_{\text{Moon}}}{d^2}$$

At the point A nearest the Moon,  $a_+ = G \frac{M_M}{(d-r)^2}$

At the point B farthest from the Moon,  $a_- = G \frac{M_M}{(d+r)^2}$

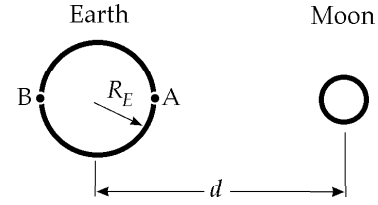


FIG. P13.48

$$\Delta a = a_+ - a = GM_M \left[ \frac{1}{(d-r)^2} - \frac{1}{d^2} \right]$$

For  $d \gg r$ ,  $\Delta a = \frac{2GM_M r}{d^3} = 1.11 \times 10^{-6} \text{ m/s}^2$

Across the planet,  $\frac{\Delta g}{g} = \frac{2\Delta a}{g} = \frac{2.22 \times 10^{-6} \text{ m/s}^2}{9.80 \text{ m/s}^2} = \boxed{2.26 \times 10^{-7}}$

**\*P13.49** Energy conservation for the two-sphere system from release to contact:

$$-\frac{Gmm}{R} = -\frac{Gmm}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$Gm \left( \frac{1}{2r} - \frac{1}{R} \right) = v^2 \quad v = \left( Gm \left[ \frac{1}{2r} - \frac{1}{R} \right] \right)^{1/2}$$

(a) The injected impulse is the final momentum of each sphere,

$$mv = m^{2/2} \left( Gm \left[ \frac{1}{2r} - \frac{1}{R} \right] \right)^{1/2} = \left[ Gm^3 \left( \frac{1}{2r} - \frac{1}{R} \right) \right]^{1/2}$$

(b) If they now collide elastically each sphere reverses its velocity to receive impulse

$$mv - (-mv) = 2mv = \boxed{2 \left[ Gm^3 \left( \frac{1}{2r} - \frac{1}{R} \right) \right]^{1/2}}$$

**P13.50** Momentum is conserved:

$$m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f}$$

$$0 = M \mathbf{v}_{1f} + 2M \mathbf{v}_{2f}$$

$$\mathbf{v}_{2f} = -\frac{1}{2} \mathbf{v}_{1f}$$

Energy is conserved:

$$(K+U)_i + \Delta E = (K+U)_f$$

$$0 - \frac{Gm_1 m_2}{r_i} + 0 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 - \frac{Gm_1 m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2} M v_{1f}^2 + \frac{1}{2} (2M) \left( \frac{1}{2} v_{1f} \right)^2 - \frac{GM(2M)}{4R}$$

$$v_{1f} = \boxed{\frac{2}{3} \sqrt{\frac{GM}{R}}} \quad v_{2f} = \frac{1}{2} v_{1f} = \boxed{\frac{1}{3} \sqrt{\frac{GM}{R}}}$$

**P13.51** (a)  $a_c = \frac{v^2}{r}$   $a_c = \frac{(1.25 \times 10^6 \text{ m/s})^2}{1.53 \times 10^{11} \text{ m}} = \boxed{10.2 \text{ m/s}^2}$

(b)  $\text{diff} = 10.2 - 9.90 = 0.312 \text{ m/s}^2 = \frac{GM}{r^2}$

$$M = \frac{(0.312 \text{ m/s}^2)(1.53 \times 10^{11} \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = \boxed{1.10 \times 10^{32} \text{ kg}}$$

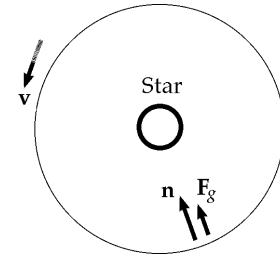


FIG. P13.51

**P13.52** (a) The free-fall acceleration produced by the Earth is  $g = \frac{GM_E}{r^2} = GM_E r^{-2}$  (directed downward)

Its rate of change is  $\frac{dg}{dr} = GM_E(-2)r^{-3} = -2GM_E r^{-3}$ .

The minus sign indicates that  $g$  decreases with increasing height.

At the Earth's surface,

$$\boxed{\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}}.$$

(b) For small differences,

$$\frac{|\Delta g|}{\Delta r} = \frac{|\Delta g|}{h} = \frac{2GM_E}{R_E^3}$$

Thus,

$$\boxed{|\Delta g| = \frac{2GM_E h}{R_E^3}}$$

(c)  $|\Delta g| = \frac{2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(6.00 \text{ m})}{(6.37 \times 10^6 \text{ m})^3} = \boxed{1.85 \times 10^{-5} \text{ m/s}^2}$

**\*P13.53** (a) Each bit of mass  $dm$  in the ring is at the same distance from the object at A. The separate contributions  $-\frac{Gmdm}{r}$  to the system energy add up to  $-\frac{GmM_{\text{ring}}}{r}$ . When the object is at A, this is

$$\frac{-6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot 1\,000 \text{ kg} \cdot 2.36 \times 10^{20} \text{ kg}}{\text{kg}^2 \sqrt{(1 \times 10^8 \text{ m})^2 + (2 \times 10^8 \text{ m})^2}} = \boxed{-7.04 \times 10^4 \text{ J}}.$$

(b) When the object is at the center of the ring, the potential energy is

$$-\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot 1\,000 \text{ kg} \cdot 2.36 \times 10^{20} \text{ kg}}{\text{kg}^2 1 \times 10^8 \text{ m}} = \boxed{-1.57 \times 10^5 \text{ J}}.$$

(c) Total energy of the object-ring system is conserved:

$$(K + U_g)_A = (K + U_g)_B$$

$$0 - 7.04 \times 10^4 \text{ J} = \frac{1}{2} 1\,000 \text{ kg} v_B^2 - 1.57 \times 10^5 \text{ J}$$

$$v_B = \left( \frac{2 \times 8.70 \times 10^4 \text{ J}}{1\,000 \text{ kg}} \right)^{1/2} = \boxed{13.2 \text{ m/s}}$$

**P13.54** To approximate the height of the sulfur, set

$$\frac{mv^2}{2} = mg_{lo}h \quad h = 70\,000 \text{ m} \quad g_{lo} = \frac{GM}{r^2} = 1.79 \text{ m/s}^2$$

$$v = \sqrt{2g_{lo}h} \quad v = \sqrt{2(1.79)(70\,000)} \approx 500 \text{ m/s (over 1 000 mi/h)}$$

A more precise answer is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\frac{1}{2}v^2 = (6.67 \times 10^{-11})(8.90 \times 10^{22}) \left( \frac{1}{1.82 \times 10^6} - \frac{1}{1.89 \times 10^6} \right) \quad v = \boxed{492 \text{ m/s}}$$

**P13.55** From the walk,  $2\pi r = 25\,000 \text{ m}$ . Thus, the radius of the planet is  $r = \frac{25\,000 \text{ m}}{2\pi} = 3.98 \times 10^3 \text{ m}$

From the drop:  $\Delta y = \frac{1}{2}gt^2 = \frac{1}{2}g(29.2 \text{ s})^2 = 1.40 \text{ m}$

so,  $g = \frac{2(1.40 \text{ m})}{(29.2 \text{ s})^2} = 3.28 \times 10^{-3} \text{ m/s}^2 = \frac{MG}{r^2} \quad \therefore M = \boxed{7.79 \times 10^{14} \text{ kg}}$

**\*P13.56** The distance between the orbiting stars is  $d = 2r \cos 30^\circ = \sqrt{3}r$  since  $\cos 30^\circ = \frac{\sqrt{3}}{2}$ . The net inward force on one orbiting star is

$$\frac{Gmm}{d^2} \cos 30^\circ + \frac{GMm}{r^2} + \frac{Gmm}{d^2} \cos 30^\circ = \frac{mv^2}{r}$$

$$\frac{Gm2 \cos 30^\circ}{3r^2} + \frac{GM}{r^2} = \frac{4\pi^2 r^2}{rT^2}$$

$$G \left( \frac{m}{\sqrt{3}} + M \right) = \frac{4\pi^2 r^3}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{G \left( M + \frac{m}{\sqrt{3}} \right)}$$

$$T = 2\pi \left( \frac{r^3}{G \left( M + \frac{m}{\sqrt{3}} \right)} \right)^{1/2}$$

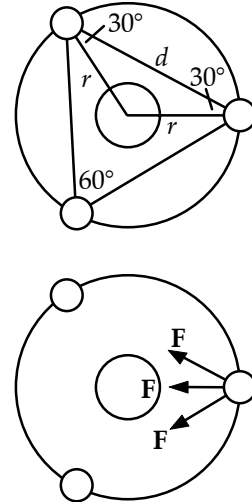


FIG. P13.56

**P13.57** For a 6.00 km diameter cylinder,  $r = 3\,000 \text{ m}$  and to simulate  $1g = 9.80 \text{ m/s}^2$

$$g = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{g}{r}} = \boxed{0.0572 \text{ rad/s}}$$

The required rotation rate of the cylinder is  $\boxed{\frac{1 \text{ rev}}{110 \text{ s}}}$

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today*, Sept. 1974.)

**P13.58** (a)  $G$  has units  $\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} = \frac{\text{kg} \cdot \text{m} \cdot \text{m}^2}{\text{s}^2 \cdot \text{kg}^2} = \frac{\text{m}^3}{\text{s}^2 \cdot \text{kg}}$

and dimensions  $[G] = \frac{\text{L}^3}{\text{T}^2 \cdot \text{M}}$ .

The speed of light has dimensions of  $[c] = \frac{\text{L}}{\text{T}}$ , and Planck's constant has the same dimensions as angular momentum or  $[h] = \frac{\text{M} \cdot \text{L}^2}{\text{T}}$ .

We require  $[G^p c^q h^r] = \text{L}$ , or  $\text{L}^{3p} \text{T}^{-2p} \text{M}^{-p} \text{L}^q \text{T}^{-q} \text{M}^r \text{L}^{2r} \text{T}^{-r} = \text{L}^1 \text{M}^0 \text{T}^0$ .

Thus,  $3p + q + 2r = 1$

$-2p - q - r = 0$

$-p + r = 0$

which reduces (using  $r = p$ ) to  $3p + q + 2p = 1$

$-2p - q - p = 0$

These equations simplify to  $5p + q = 1$  and  $q = -3p$ .

Then,  $5p - 3p = 1$ , yielding  $p = \frac{1}{2}$ ,  $q = -\frac{3}{2}$ , and  $r = \frac{1}{2}$ .

Therefore, Planck length =  $\boxed{G^{1/2} c^{-3/2} h^{1/2}}$ .

(b)  $\left(6.67 \times 10^{-11}\right)^{1/2} \left(3 \times 10^8\right)^{-3/2} \left(6.63 \times 10^{-34}\right)^{1/2} = \left(1.64 \times 10^{-69}\right)^{1/2} = 4.05 \times 10^{-35} \text{ m} \boxed{\sim 10^{-34} \text{ m}}$

**P13.59**  $\frac{1}{2} m_0 v_{\text{esc}}^2 = \frac{G m_p m_0}{R}$

$v_{\text{esc}} = \sqrt{\frac{2Gm_p}{R}}$

With  $m_p = \rho \frac{4}{3} \pi R^3$ , we have

$$\begin{aligned} v_{\text{esc}} &= \sqrt{\frac{2G\rho \frac{4}{3} \pi R^3}{R}} \\ &= \sqrt{\frac{8\pi G\rho}{3} R} \end{aligned}$$

So,  $v_{\text{esc}} \propto R$ .

**\*P13.60** For both circular orbits,

$$\sum F = ma:$$

$$\frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

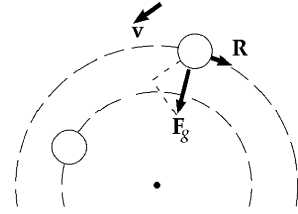


FIG. P13.60

(a) The original speed is  $v_i = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m} + 2 \times 10^5 \text{ m})}} = \boxed{7.79 \times 10^3 \text{ m/s}}.$

(b) The final speed is  $v_f = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.47 \times 10^6 \text{ m})}} = \boxed{7.85 \times 10^3 \text{ m/s}}.$

The energy of the satellite-Earth system is

$$K + U_g = \frac{1}{2}mv^2 - \frac{GM_E m}{r} = \frac{1}{2}m \frac{GM_E}{r} - \frac{GM_E m}{r} = -\frac{GM_E m}{2r}$$

(c) Originally  $E_i = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(100 \text{ kg})}{2(6.57 \times 10^6 \text{ m})} = \boxed{-3.04 \times 10^9 \text{ J}}.$

(d) Finally  $E_f = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(100 \text{ kg})}{2(6.47 \times 10^6 \text{ m})} = \boxed{-3.08 \times 10^9 \text{ J}}.$

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i - E_f = -3.04 \times 10^9 \text{ J} - (-3.08 \times 10^9 \text{ J}) = \boxed{4.69 \times 10^7 \text{ J}}.$$

(f) The only forces on the object are the backward force of air resistance  $R$ , comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force, one component of the gravitational force pulls forward on the satellite to do positive work and make its speed increase.

- P13.61** (a) At infinite separation  $U = 0$  and at rest  $K = 0$ . Since energy of the two-planet system is conserved we have,

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \quad (1)$$

The initial momentum of the system is zero and momentum is conserved.

Therefore,  $0 = m_1v_1 - m_2v_2 \quad (2)$

Combine equations (1) and (2): 
$$v_1 = m_2 \sqrt{\frac{2G}{d(m_1 + m_2)}} \quad \text{and} \quad v_2 = m_1 \sqrt{\frac{2G}{d(m_1 + m_2)}}$$

Relative velocity 
$$v_r = v_1 - (-v_2) = \sqrt{\frac{2G(m_1 + m_2)}{d}}$$

- (b) Substitute given numerical values into the equation found for  $v_1$  and  $v_2$  in part (a) to find

$$v_1 = 1.03 \times 10^4 \text{ m/s} \quad \text{and} \quad v_2 = 2.58 \times 10^3 \text{ m/s}$$

Therefore,  $K_1 = \frac{1}{2}m_1v_1^2 = \boxed{1.07 \times 10^{32} \text{ J}}$  and  $K_2 = \frac{1}{2}m_2v_2^2 = \boxed{2.67 \times 10^{31} \text{ J}}$

- P13.62** (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$$mr_a v_a = mr_p v_p \text{ and } v_a = v_p \left( \frac{r_p}{r_a} \right) = (3.027 \times 10^4 \text{ m/s}) \left( \frac{1.471}{1.521} \right) = \boxed{2.93 \times 10^4 \text{ m/s}}$$

(b)  $K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}(5.98 \times 10^{24})(3.027 \times 10^4)^2 = \boxed{2.74 \times 10^{33} \text{ J}}$

$$U_p = -\frac{GmM}{r_p} = -\frac{(6.673 \times 10^{-11})(5.98 \times 10^{24})(1.99 \times 10^{30})}{1.471 \times 10^{11}} = \boxed{-5.40 \times 10^{33} \text{ J}}$$

(c) Using the same form as in part (b),  $K_a = \boxed{2.57 \times 10^{33} \text{ J}}$  and  $U_a = \boxed{-5.22 \times 10^{33} \text{ J}}$ .

Compare to find that  $K_p + U_p = \boxed{-2.66 \times 10^{33} \text{ J}}$  and  $K_a + U_a = \boxed{-2.65 \times 10^{33} \text{ J}}$ . They agree.

**P13.63** (a) The work must provide the increase in gravitational energy

$$\begin{aligned}
 W &= \Delta U_g = U_{gf} - U_{gi} \\
 &= -\frac{GM_E M_p}{r_f} + \frac{GM_E M_p}{r_i} \\
 &= -\frac{GM_E M_p}{R_E + y} + \frac{GM_E M_p}{R_E} \\
 &= GM_E M_p \left( \frac{1}{R_E} - \frac{1}{R_E + y} \right) \\
 &= \left( \frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2}{\text{kg}^2} \right) (5.98 \times 10^{24} \text{ kg}) (100 \text{ kg}) \left( \frac{1}{6.37 \times 10^6 \text{ m}} - \frac{1}{7.37 \times 10^6 \text{ m}} \right) \\
 W &= \boxed{850 \text{ MJ}}
 \end{aligned}$$

(b) In a circular orbit, gravity supplies the centripetal force:

$$\frac{GM_E M_p}{(R_E + y)^2} = \frac{M_p v^2}{(R_E + y)}$$

$$\text{Then, } \frac{1}{2} M_p v^2 = \frac{1}{2} \frac{GM_E M_p}{(R_E + y)}$$

So, additional work = kinetic energy required

$$\begin{aligned}
 &= \frac{1}{2} \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2)(5.98 \times 10^{24} \text{ kg})(100 \text{ kg})}{(\text{kg}^2)(7.37 \times 10^6 \text{ m})} \\
 \Delta W &= \boxed{2.71 \times 10^9 \text{ J}}
 \end{aligned}$$

**P13.64** Centripetal acceleration comes from gravitational acceleration.

$$\begin{aligned}
 \frac{v^2}{r} &= \frac{M_c G}{r^2} = \frac{4\pi^2 r}{T^2} \\
 GM_c T^2 &= 4\pi^2 r^3 \\
 (6.67 \times 10^{-11})(20)(1.99 \times 10^{30})(5.00 \times 10^{-3})^2 &= 4\pi^2 r^3 \\
 r_{\text{orbit}} &= \boxed{119 \text{ km}}
 \end{aligned}$$

$$\text{P13.65 (a) } T = \frac{2\pi r}{v} = \frac{2\pi(30\,000 \times 9.46 \times 10^{15} \text{ m})}{2.50 \times 10^5 \text{ m/s}} = 7 \times 10^{15} \text{ s} = \boxed{2 \times 10^8 \text{ yr}}$$

$$\text{(b) } M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2 (30\,000 \times 9.46 \times 10^{15} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.13 \times 10^{15} \text{ s})^2} = 2.66 \times 10^{41} \text{ kg}$$

$$M = 1.34 \times 10^{11} \text{ solar masses} \quad \boxed{\sim 10^{11} \text{ solar masses}}$$

The number of stars is  $\boxed{\text{on the order of } 10^{11}}$ .

- P13.66** (a) From the data about perigee, the energy of the satellite-Earth system is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)(8.23 \times 10^3)^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)}{7.02 \times 10^6}$$

or  $E = \boxed{-3.67 \times 10^7 \text{ J}}$

(b)  $L = mvr \sin \theta = mv_p r_p \sin 90.0^\circ = (1.60 \text{ kg})(8.23 \times 10^3 \text{ m/s})(7.02 \times 10^6 \text{ m})$   
 $= \boxed{9.24 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}}$

- (c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have  $\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$

and  $mv_a r_a \sin 90.0^\circ = L.$

Thus,  $\frac{1}{2}(1.60)v_a^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)}{r_s} = -3.67 \times 10^7 \text{ J}$

and  $(1.60 \text{ kg})v_a r_a = 9.24 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}.$

Solving simultaneously,  $\frac{1}{2}(1.60)v_a^2 - \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(1.60)(1.60)v_a}{9.24 \times 10^{10}} = -3.67 \times 10^7$

which reduces to  $0.800v_a^2 - 11\,046v_a + 3.6723 \times 10^7 = 0$

so  $v_a = \frac{11\,046 \pm \sqrt{(11\,046)^2 - 4(0.800)(3.6723 \times 10^7)}}{2(0.800)}.$

This gives  $v_a = 8\,230 \text{ m/s}$  or  $\boxed{5\,580 \text{ m/s}}$ . The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus,  $r_a = \frac{L}{mv_a} = \frac{9.24 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}}{(1.60 \text{ kg})(5.58 \times 10^3 \text{ m/s})} = \boxed{1.04 \times 10^7 \text{ m}}.$

- (d) The major axis is  $2a = r_p + r_a$ , so the semi-major axis is

$$a = \frac{1}{2}(7.02 \times 10^6 \text{ m} + 1.04 \times 10^7 \text{ m}) = \boxed{8.69 \times 10^6 \text{ m}}$$

(e)  $T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} = \sqrt{\frac{4\pi^2 (8.69 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}}$

$T = 8\,060 \text{ s} = \boxed{134 \text{ min}}$

- \*P13.67** Let  $m$  represent the mass of the meteoroid and  $v_i$  its speed when far away. No torque acts on the meteoroid, so its angular momentum is conserved as it moves between the distant point and the point where it grazes the Earth, moving perpendicular to the radius:

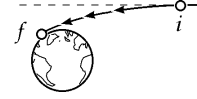


FIG. P13.67

$$\begin{aligned} L_i &= L_f: & m\mathbf{r}_i \times \mathbf{v}_i &= m\mathbf{r}_f \times \mathbf{v}_f \\ & & m(3R_E v_i) &= mR_E v_f \\ & & v_f &= 3v_i \end{aligned}$$

Now energy of the meteoroid-Earth system is also conserved:

$$\begin{aligned} (K + U_g)_i &= (K + U_g)_f: & \frac{1}{2}mv_i^2 + 0 &= \frac{1}{2}mv_f^2 - \frac{GM_E m}{R_E} \\ & & \frac{1}{2}v_i^2 &= \frac{1}{2}(9v_i^2) - \frac{GM_E}{R_E} \end{aligned}$$

$$\frac{GM_E}{R_E} = 4v_i^2: \quad \boxed{v_i = \sqrt{\frac{GM_E}{4R_E}}}$$

- \*P13.68** From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Figure P13.35 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius  $R$  of the planet.

$$\begin{aligned} \sum F &= ma: & \frac{GMm}{R^2} &= \frac{mv^2}{R} = \frac{m}{R} \left( \frac{2\pi R}{T} \right)^2 \\ & & G\rho V &= \frac{R^2(4\pi^2 R^2)}{RT^2} \\ & & G\rho \left( \frac{4}{3}\pi R^3 \right) &= \frac{4\pi^2 R^3}{T^2} \end{aligned}$$

The radius divides out:  $T^2 G\rho = 3\pi$

$$\boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$$

- P13.69** If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M + m} \quad \text{and} \quad Mr_2 = mr_1$$

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass  $F = ma$  so

$$mr_1\omega_1^2 = \frac{MGm}{d^2} \quad \text{and} \quad Mr_2\omega_2^2 = \frac{MGm}{d^2}$$

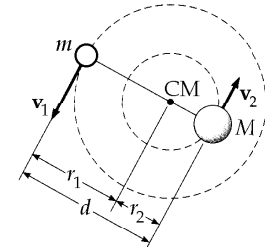


FIG. P13.69

Combining these two equations and using  $d = r_1 + r_2$  gives  $(r_1 + r_2)\omega^2 = \frac{(M + m)G}{d^2}$

with  $\omega_1 = \omega_2 = \omega$

$$\text{and } T = \frac{2\pi}{\omega}$$

$$\text{we find } \boxed{T^2 = \frac{4\pi^2 d^3}{G(M + m)}}.$$

- P13.70** (a) The gravitational force exerted on  $m_2$  by the Earth (mass  $m_1$ ) accelerates  $m_2$  according to:  $m_2 g_2 = \frac{G m_1 m_2}{r^2}$ . The equal magnitude force exerted on the Earth by  $m_2$  produces negligible acceleration of the Earth. The acceleration of relative approach is then

$$g_2 = \frac{G m_1}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(1.20 \times 10^7 \text{ m})^2} = \boxed{2.77 \text{ m/s}^2}.$$

- (b) Again,  $m_2$  accelerates toward the center of mass with  $g_2 = 2.77 \text{ m/s}^2$ . Now the Earth accelerates toward  $m_2$  with an acceleration given as

$$m_1 g_1 = \frac{G m_1 m_2}{r^2}$$

$$g_1 = \frac{G m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(2.00 \times 10^{24} \text{ kg})}{(1.20 \times 10^7 \text{ m})^2} = 0.926 \text{ m/s}^2$$

The distance between the masses closes with relative acceleration of

$$g_{\text{rel}} = g_1 + g_2 = 0.926 \text{ m/s}^2 + 2.77 \text{ m/s}^2 = \boxed{3.70 \text{ m/s}^2}.$$

**P13.71** *Initial Conditions and Constants:*

|                   |                                  |
|-------------------|----------------------------------|
| Mass of planet:   | $5.98 \times 10^{24} \text{ kg}$ |
| Radius of planet: | $6.37 \times 10^6 \text{ m}$     |
| Initial $x$ :     | 0.0 planet radii                 |
| Initial $y$ :     | 2.0 planet radii                 |
| Initial $v_x$ :   | +5 000 m/s                       |
| Initial $v_y$ :   | 0.0 m/s                          |
| Time interval:    | 10.9 s                           |

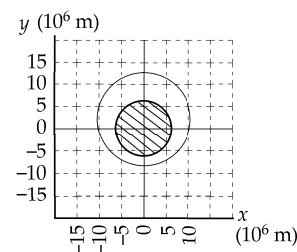


**FIG. P13.71**

| $t$ (s)  | $x$ (m)    | $y$ (m)      | $r$ (m)      | $v_x$ (m/s) | $v_y$ (m/s) | $a_x$ ( $\text{m/s}^2$ ) | $a_y$ ( $\text{m/s}^2$ ) |
|----------|------------|--------------|--------------|-------------|-------------|--------------------------|--------------------------|
| 0.0      | 0.0        | 12 740 000.0 | 12 740 000.0 | 5 000.0     | 0.0         | 0.000 0                  | -2.457 5                 |
| 10.9     | 54 315.3   | 12 740 000.0 | 12 740 115.8 | 4 999.9     | -26.7       | -0.010 0                 | -2.457 4                 |
| 21.7     | 108 629.4  | 12 739 710.0 | 12 740 173.1 | 4 999.7     | -53.4       | -0.021 0                 | -2.457 3                 |
| 32.6     | 162 941.1  | 12 739 130.0 | 12 740 172.1 | 4 999.3     | -80.1       | -0.031 0                 | -2.457 2                 |
| ...      |            |              |              |             |             |                          |                          |
| 5 431.6  | 112 843.8  | -8 466 816.0 | 8 467 567.9  | -7 523.0    | -39.9       | -0.074 0                 | 5.562 5                  |
| 5 442.4  | 31 121.4   | -8 467 249.7 | 8 467 306.9  | -7 523.2    | 20.5        | -0.020 0                 | 5.563 3                  |
| 5 453.3  | -50 603.4  | -8 467 026.9 | 8 467 178.2  | -7 522.8    | 80.9        | 0.033 0                  | 5.563 4                  |
| 5 464.1  | -132 324.3 | -8 466 147.7 | 8 467 181.7  | -7 521.9    | 141.4       | 0.087 0                  | 5.562 8                  |
| ...      |            |              |              |             |             |                          |                          |
| 10 841.3 | -108 629.0 | 12 739 134.4 | 12 739 597.5 | 4 999.9     | 53.3        | 0.021 0                  | -2.457 5                 |
| 10 852.2 | -54 314.9  | 12 739 713.4 | 12 739 829.2 | 5 000.0     | 26.6        | 0.010 0                  | -2.457 5                 |
| 10 863.1 | 0.4        | 12 740 002.4 | 12 740 002.4 | 5 000.0     | -0.1        | 0.000 0                  | -2.457 5                 |

The object does not hit the Earth; its minimum radius is  $1.33 R_E$ .

Its period is  $1.09 \times 10^4 \text{ s}$ . A circular orbit would require a speed of  $5.60 \text{ km/s}$ .

## ANSWERS TO EVEN PROBLEMS

- P13.2**  $2.67 \times 10^{-7} \text{ m/s}^2$
- P13.4** 3.00 kg and 2.00 kg
- P13.6** (a)  $4.39 \times 10^{20} \text{ N}$  toward the Sun;  
(b)  $1.99 \times 10^{20} \text{ N}$  toward the Earth;  
(c)  $3.55 \times 10^{22} \text{ N}$  toward the Sun
- P13.8** see the solution; either 1 m – 61.3 nm or  $2.74 \times 10^{-4} \text{ m}$
- P13.10**  $\frac{2}{3}$
- P13.12** (a) 1.02 km/s; (b) 1.35 mm
- P13.14** see the solution
- P13.16** 1.27
- P13.18** Planet Y has turned through 1.30 revolutions
- P13.20**  $1.63 \times 10^4 \text{ rad/s}$
- P13.22** 18.2 ms
- P13.24** (a)  $1.31 \times 10^{17} \text{ N}$  toward the center;  
(b)  $2.62 \times 10^{12} \text{ N/kg}$
- P13.26** (a)  $-4.77 \times 10^9 \text{ J}$ ; (b) 569 N down;  
(c) 569 N up
- P13.28**  $2.52 \times 10^7 \text{ m}$
- P13.30**  $2.82 \times 10^9 \text{ J}$
- P13.32** (a) see the solution; (b) 340 s
- P13.34** (a) 42.1 km/s; (b)  $2.20 \times 10^{11} \text{ m}$
- P13.36**  $1.58 \times 10^{10} \text{ J}$
- P13.38** (a)  $2\pi(R_E + h)^{3/2}(GM_E)^{-1/2}$ ;  
(b)  $(GM_E)^{1/2}(R_E + h)^{-1/2}$ ;  
(c)  $GM_E m \left[ \frac{R_E + 2h}{2R_E(R_E + h)} \right] - \frac{2\pi^2 R_E^2 m}{(86\,400 \text{ s})^2}$   
The satellite should be launched from the Earth's equator toward the east.
- P13.40** (a) 10.0 m/s<sup>2</sup>; (b) 21.8 km/s
- P13.42** 11.8 km/s
- P13.44**  $\frac{GM_E m}{12R_E}$
- P13.46** (a)  $v_0 = \left( \frac{GM_E}{r} \right)^{1/2}$ ; (b)  $v_i = \frac{5 \left( \frac{GM_E}{r} \right)^{1/2}}{4}$ ;  
(c)  $r_f = \frac{25r}{7}$
- P13.48**  $2.26 \times 10^{-7}$
- P13.50**  $\frac{2}{3} \sqrt{\frac{GM}{R}}$ ;  $\frac{1}{3} \sqrt{\frac{GM}{R}}$
- P13.52** (a), (b) see the solution;  
(c)  $1.85 \times 10^{-5} \text{ m/s}^2$
- P13.54** 492 m/s
- P13.56** see the solution
- P13.58** (a)  $G^{1/2} c^{-3/2} h^{1/2}$ ; (b)  $\sim 10^{-34} \text{ m}$
- P13.60** (a) 7.79 km/s; (b) 7.85 km/s; (c) -3.04 GJ;  
(d) -3.08 GJ; (e) loss = 46.9 MJ;  
(f) A component of the Earth's gravity pulls forward on the satellite in its downward banking trajectory.
- P13.62** (a) 29.3 km/s; (b)  $K_p = 2.74 \times 10^{33} \text{ J}$ ;  
 $U_p = -5.40 \times 10^{33} \text{ J}$ ; (c)  $K_a = 2.57 \times 10^{33} \text{ J}$ ;  
 $U_a = -5.22 \times 10^{33} \text{ J}$ ; yes
- P13.64** 119 km
- P13.66** (a) -36.7 MJ; (b)  $9.24 \times 10^{10} \text{ kg} \cdot \text{m}^2/\text{s}$ ;  
(c) 5.58 km/s; 10.4 Mm; (d) 8.69 Mm;  
(e) 134 min
- P13.68** see the solution
- P13.70** (a) 2.77 m/s<sup>2</sup>; (b) 3.70 m/s<sup>2</sup>